Mathematics: Keystone to Student Learning

> Have you noticed, too, how people with a talent for calculation are naturally quick at learning almost any subject; and how a training in it makes a slow mind quicker, even if it does no other good? - Plato, *The Republic*[1]

Introduction

In 1993-1995 two of the authors developed and implemented an instructional program for remedial mathematics students, and in collaboration with a third colleague, studied its effects. The study looked at the resulting outcomes in three areas: retention, student performance in mathematics, and students' generalized study skills. The results of this study are reported in (Sagher, Siadat, & Hagedorn, 2000).

One shortcoming of the study was that the evaluation of students' content knowledge, although very convincing, could not be established by a direct comparison with an equivalent group of students on a common final exam. Another shortcoming was that the program was carried out by only one teacher. In 1998, the Gabriella and Paul Rosenbaum Foundation funded an expansion of the study. This report summarizes the outcomes of the expanded project in its first year, 1998-1999. We report on direct comparisons on common final exams, and on the performance of students taught by three teachers using the program's methods, and four teachers for the control classes. As in the original study, we also measured the effects of the program on generalized study skills.

The results indicate dramatically improved student performance and retention in elementary algebra, intermediate algebra, and college algebra classes. Additionally, students in these classes also performed better on standardized arithmetic and reading comprehension tests. The improvement on these tests may reflect students' enhanced ability to integrate their intellectual resources and to focus their attention.

The Need for the Project

Many professions have important mathematics prerequisites; others use mathematics as a filter. Mathematics is, therefore, a limiting factor for the professional aspirations of a large number of people.

Remedial mathematics is a large part of the teaching program of post-secondary institutions. It presents a great challenge both to students and teachers. Students have to correct the causes of past failures in mathematics, burdened by a diminished self-confidence. Teachers need to address students' weaknesses, without causing further damage to their self-confidence.

All institutions of higher learning offer remedial programs, particularly in mathematics (Mansfield, et al., 1991; Adelman, 1995). Young (1993) provides data showing that more than 15 percent of mathematics registrations in colleges and universities in the U.S. in fall 1990 were in remedial classes, and more than 50 percent were in courses below calculus. A 1995 study by the U.S. Department of Education, National Center for Education Statistics, on remedial courses found that the greatest need for catch-up classes was in math, taken by 24 percent of all first time freshmen, regardless of whether they attended a two- or four-year school (Naylor, 1997). Even at leading mathematics research departments in the country, remedial mathematics constituted 9 percent of undergraduate enrollments in mathematics in fall 1995 (Fulton, 1996).

At the community college level the need for remedial/developmental programs is even greater. Enrollments in remedial courses at two-year institutions climbed from a third of the total mathematics program enrollment in 1970 to more than half in 1990. In this period, "remediation was classified as a major problem by 65 percent of [mathematics] department heads" (Watkins et al., 1993, p.56).

At the college in which this study was conducted, remedial mathematics constituted 62 percent of all mathematics registration in the fall 1995 semester and more than 70 percent in fall 1999. Our study shows that the problem of remedial education may be much deeper than is generally realized. Young (1993) writes "Remedial mathematics does not remediate; few people taking college algebra complete more than one semester of calculus." The problem seems to be that few people taking college algebra actually learn college algebra. For example, if we define the yield of a class as the percent of students who score 70 percent or above on the final exam, the yields for traditional teaching in our study are around 10 percent.

Methodology

The underlying assumption of the original study, and its successor, the Keystone project, is that unsuccessful students do not necessarily lack the intelligence or the desire to succeed. Rather, they are held back by behavior patterns which inhibit learning. Findings of several different authors on the reasons for weak academic performance were combined in the following list, provided in (Sagher, Siadat, & Hagedorn, 2000).

- Students' short attention spans (Horn & Packard, 1985; Soraci, Jr., et al., 1986; Lee & Meyer, 1994)
- Inadequate attention to assigned homework (Keith, 1982; Vratanina, 1988; Robinson, 1994)
- Short time horizons (Solomon & Rothblum, 1984; Beswick, Rothblum & Mann, 1988)
- Failure to learn from errors (Hodges, 1981)
- Passivity in class, hoping to pass unnoticed (Hodges, 1981; Lee & Meyer, 1994)
- Poor attendance (de Jung & Duckworth, 1985, 1986)
- Low self-esteem (Gill, 1969; Meyer, 1972; Calsyn & Kenny, 1977; Carr, Barkowski, & Maxwell, 1991)
- Ignoring teachers' statements

The Keystone approach targets these reasons for inadequate performance and provides inter-linked interventions to address each.

To address short attention spans, the instructor gives short, timed-pressured quizzes that require student's full concentration. These quizzes are based on the homework and so motivate completion of homework assignments. Students are quizzed at each class meeting, presenting them with deadlines within their short time-horizons. To help students learn from mistakes, questions on which students have a low rate of success are repeated on follow-up quizzes. To combat passivity, the instructor uses cooperative learning. To address poor attendance, students who miss more than three class periods are dropped from the class. Cumulative quizzes and tests give students an opportunity to demonstrate mastery where they have had failure, building student's self-confidence. To teach attentiveness to the instructor, students receive repeated evidence that following instructions produces success.

Finally, since relative grading (grading on a curve) is antithetical to cooperation as well as to striving for true excellence, students in the project are graded on an absolute scale. Most of the students have work experience. To help them gain an understanding of the underlying principles of our expectations of them, they are told to apply to the classroom the standards of a workplace: a strict attendance code, quick and accurate completion of assigned work with immediate feedback, and evaluation of performance in accord with absolute, rather than relative, standards.

Educational research indicates that frequent testing, which is an essential tool of the project, benefits the learner: it encourages regular study habits, discourages cramming, and mitigates test anxiety (Mawhinney, et al., 1971; Dempster, 1992). In addition, it builds upon findings that cumulative tests are highly effective in consolidating student learning (Dempster, 1992).

Classroom Management

Both project and control classes meet twice a week for one hundred minutes per session. Keystone project classes begin with a short period of question/answer followed by a homework-based multiple-choice quiz.[2] Computer scoring provides an item analysis and statistical information on each problem for guidance in planning for the next class period and quiz. The instructor either speeds up or slows down the pace of instruction based on the results. When the standard deviation of the scores exceeds 25%, indicating that the class is splitting, cooperative learning is employed. When the standard deviation recedes to a more acceptable level, cooperative learning is suspended, and the teacher accelerates the pace of instruction.

Assessment of the Results and Effectiveness

The 1998-99 results encompass eleven project classes taught by three teachers and nine control classes taught by four other teachers. There were initially 332 students in the project classes and 311 in the control classes. Each student chose his or her class without knowing whether the class was project or control. The project and control classes were in three subjects: Elementary Algebra (Math 110), Intermediate Algebra (Math 112), and College Algebra (Math 140).

To measure the effects of the program on generalized educational skills, in particular concentration skills, we administered pre- and post-test College Board "Descriptive Tests in Mathematics Skills in Arithmetic Skills" and "Descriptive Tests of Language Skills in Reading Comprehension" (forms K and L) to all students.

Figure 1 presents the gains in College Board's Arithmetic Tests for the project and the control classes. The improvement of students in project classes was about double that of students in control classes. Since arithmetic is a prerequisite subject for all algebra classes, the improvement may be attributed to improved concentration skills.
See book's website *http://www.rapidintellect.com/SI-2001.htm*

Figure 2 presents the change in College Board's Reading Comprehension Tests scores for project and the control classes. No language skills were taught in the project classes. We interpret the marked difference in the improvement of reading comprehension between students in the project classes and students in the control classes as a manifestation of improved study skills, most likely improved concentration skills.

This finding is fully in line with the results reported in the previous study (Sagher, Siadat, & Hagedorn, 2000). They provide experimental verification of Plato's observation, made some 2400 years ago.

See book's website *http://www.rapidintellect.com/SI-2001.htm*

To assess the effects of the intervention on the learning of mathematics, we administered a common final exam to project and to control classes. In Figure 3 we report the mean final exam scores for the project and control classes. Combining the results of the three courses, we get that students in the project classes scored more than twenty percentage points, two letter grades, higher than students in the control classes.
See book's website *http://www.rapidintellect.com/SI-2001.htm*

The results reported in Figure 3 should be read in conjunction with the retention rates: it is possible to raise average performance on the final exam by encouraging weaker students to drop the class. We present the retention rates for project and control classes in Figure 4. The retention figures show that the higher performance was achieved with higher retention.
See book's website *http://www.rapidintellect.com/SI-2001.htm*

Another important measure of effectiveness of the program is the yield of a class as defined in a previous section, the percentage of students who score seventy percent or better on the final exam, showing that they are ready to take the next mathematics class. These outcomes for project and control classes are reported in Figure 5.
See book's website *http://www.rapidintellect.com/SI-2001.htm*

Applicability to Other Institutions

The Keystone Project was the winner of the 1999 National Council of Instructional Administrators "Exemplary Initiative award for classroom learning." The award recognizes not just the results achieved in one institution, but expresses recognition that the method is generally applicable. Moreover, the Keystone Project was featured in the lead article of the January 2000 issue of *Teaching for Success*, the largest monthly faculty development publication in the country.

The methods of the Keystone project are applicable to all institutions facing the problem of remedial education in mathematics. The initial cost of computers, software, and a scanning device are well within the annual technology budgets of most institutions. Faculty would need to be trained and, in the initial phases, compensated for the additional time required to write a larger than usual number of multiple choice quizzes and tests. However, these test items can be saved and re-used in future years reducing the workload of faculty using the methods of the project to normal levels.

In any case, the dramatically higher yields of students prepared for the next courses easily justify the modest investment of additional resources. Investing repeatedly in the education of students who, in large numbers, repeat remedial courses taught in the traditional way, is the most expensive way of helping these students.

We should like to warn, however, against a reductionist approach to the method. It is not about more frequent examinations, or absolute, rather than relative, standards of performance, or about tougher policies of attendance. It is an integrated approach, and if adopted, should be adopted as a whole.

Conclusion

The keystone is the essential piece in an arch: the stone that holds all others in place. The Keystone Project demonstrates that mathematics can serve as a "keystone" in the education of students needing remediation. As we have seen, proper remediation in mathematics results in a large improvement in student performance not only in mathematics, but also in reading comprehension. Plato was right.

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