

## Keystone Method: A Learning Paradigm in Mathematics

M. Vali Siadat, Paul M. Musial, and Yoram Sagher

**Abstract:** This study reports the effects of an integrated instructional program (the Keystone Method) on the students' performance in mathematics and reading, and tracks students' persistence and retention. The subject of the study was a large group of students in remedial mathematics classes at the college, willing to learn but lacking basic educational skills. The results show not only improvements in student outcomes in mathematics, but also gains in reading comprehension scores, as compared with the control group. These results were achieved at no cost to classroom retention. The persistence rates of the Keystone students were also higher for the subsequent terms.

**Keywords:** Keystone Method, dynamic assessment, immediate feedback, cooperative learning, peer tutoring, acquisition of mastery, concentration skills, persistence, classroom retention, automaticity.

### 1. INTRODUCTION

The Keystone Project at Daley College has its genesis in 1993 when a major study investigated the causes of student failure in mathematics and devised an instructional plan to address those difficulties. The study targeted three areas for improvement: student performance in mathematics, classroom retention, and students' concentration skills. The results of the study are reported in [19, 20]. The research was subsequently expanded in 1998–1999 to encompass more classes with the involvement of more teachers through a grant from Gabriella and Paul Rosenbaum Foundation [21]. The present report summarizes the cumulative results for 1998–2000 and also the outcomes of the expanded project for the next two years, 2001–2003. We report on direct comparisons on mathematics common final exams, and on the reading comprehension tests. We also report the persistence of Keystone

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students in mathematics and also at the college. The present results corroborate the earlier studies and show strong student gains in basic, intermediate, and college algebra classes as well as improvement in reading comprehension scores. The latter effect is attributed to the students' improved concentration skills.

## 2. CONCEPTUAL FRAMEWORK

The theoretical framework of the Keystone method is based on current educational research on learning, psychology, and causes of student failure in mathematics. It identifies a set of behavior habits which inhibit student learning. These include

- a) students' short attention spans [11, 13, 23]
- b) ignoring homework assignments [12, 18, 24]
- c) short time horizons [1, 22]
- d) failure to learn from mistakes [10]
- e) passivity in class [10, 13]
- f) poor attendance patterns [5, 6]
- g) low self esteem [2, 3, 9, 15]
- h) inattention to teacher's statements [17].

The Keystone approach targets these reasons for failure through an instructional regimen which utilizes dynamic assessment of student learning. To address short attention spans, the instructor administers a short, time-pressured test at the beginning of each class. The tests are all cumulative and teach students to integrate their knowledge of the material and to work fast and accurately, with full concentration. Research has shown that frequent testing inculcates regular and steady study habits, discourages cramming and mitigates test anxiety [7, 14]. Moreover, cumulative testing motivates students to constantly review the earlier topics and concepts and plays an important role in consolidating student learning [7, 16].

To encourage completing the homework assignments, the instructor rewards students on the homework-based tests. To alleviate the learning problems caused by the students' short time horizons, the instructor gives frequent deadlines within those horizons. To help students learn from their mistakes, the instructor repeats questions on which the class average is low on follow-up tests until mastery is achieved. To combat passivity, the instructor incorporates cooperative learning and peer tutoring into his lectures. To address poor attendance, the instructor issues administrative drops (instructor-initiated withdrawals) following three absences, only to be reversed upon the student meeting certain conditions. To improve student's

self-esteem, the instructor gives well-defined tasks, immediate feedback, and repeated evidence of success on the tests. To teach attentiveness to instructor's statements, the student receives actual evidence that following instruction produces success.

In short, students enter into a continuous dialogue with the teacher through a specific medium—the tests—where the teacher conveys his/her expectation and policies, and students respond by their performance. Student performance, on the other hand, provides a vital feedback to the teacher on the basis of which he/she could adjust the pace and content of instruction. The tests are mostly multiple choice but also contain some open-ended questions to be corrected by the teacher. The multiple choice questions are carefully designed and provide for statistical analysis of the entire test as well as item analysis of each question. Grading of all tests are decided on an absolute scale, requiring each student to acquire a certain level of mastery, irrespective of other students' scores. Thus, cooperation and striving for excellence is promoted among all students without anyone's fear of being graded relative to others.

An essential element of the Keystone method is improving the student's ability to work with full concentration. When a student works with good concentration, he/she will have a better recall of the facts that are part of the work on the task. It also improves the student's performance of the task. We build the student's improved ability to concentrate by the administration of time-pressured tests. To do well on those tests the student has to focus his/her attention fully for the duration of the time allotted for the test.

Time-pressured tests also demand that students develop a mastery of basic skills. Prompt feedbacks facilitate this development. Through attainment of mastery of such skills as performing basic arithmetic operations, using algebraic procedures, recalling basic mathematical facts, etc., these skills are automatized. Students can recall the basic facts without the diversion of attentional resources. Development of automaticity in basic skills allows the students to move up to the higher levels of thought as it frees up attentional resources to work on more difficult problems [26]. "When cognitive processes (e.g., reading, writing grammatically, . . . , using simple mathematical procedures) become automatic, they demand very little space in working memory, they occur rapidly, and they often occur without conscious effort." [25]

At the start of the semester the tests are closely related to the homework, establishing regular work habits in the students. As the semester progresses the tests become longer and cumulative. The longer tests develop the student's ability to work longer with full concentration. The cumulative nature of the tests inculcates habits of constant review of previous material.

We require excellence from all. Contrary to a common belief that attributes development of high level of skills to innate ability, it is possible

to achieve expertise through sustained practice and concentration on the task. As pointed out in [8], experts themselves attribute their success in attainment of higher levels of skills to practice and the ability to maintain concentration during long practice sessions.

### 3. THE KEYSTONE METHOD IN THE CLASSROOM

Application of the Keystone method in the classroom involves three elements: daily assessment of students, use of feedback by the teacher, and development of cooperative learning groups. At the beginning of each Keystone class the teacher distributes a student-by-student performance report for the entire class roster (with individual student identities masked). The daily performance (roster) report reflects each student's progress on quizzes and tests, as well as the performance of the entire class. This report is both a form of accountability and a motivational tool. As an absence from class constitutes a zero score on the missed test, the report underlines for students the correlation between attendance and performance. This daily roster report is a highly effective means of communication between the teacher and the students: as students know where they stand at all times, misperception becomes impossible.

The class period proceeds with a short question/answer session, following which the teacher administers a homework-based, time-pressured, cumulative quiz. The cumulative aspect of the quiz is important: the difficulties of weaker students are identified, mastery is continuously rewarded, and self-esteem is heightened. After the quiz, the teacher provides immediate feedback by reviewing the quiz questions and explaining the difficult problems.

Following the class period, the teacher performs statistical analysis of the quiz scores and an item analysis of student performance on each question. This analysis provides not only a global knowledge of class performance, but also feedback on which types of problems to include on future quizzes. The teacher will then re-teach troublesome problems and replicates them in subsequent quizzes so that students attain a level of mastery. This technique of "ironing out" areas in which the class has difficulty is highly effective.

But there is also a backup plan. When standard deviation of quiz scores exceeds 25%, indicating a serious split in the skills level, the teacher moves from lecture method to cooperative learning and peer tutoring. In this mode, the teacher divides the students into small groups and becomes a facilitator, moving between groups. These cooperative learning groups bring together students from each quartile of the class according to their overall scores. In such settings the weaker students learn from the stronger students, and the stronger students benefit by reinforcing their own knowledge.

Interaction of weaker and stronger students within groups has a social dimension also—it mitigates passivity and promotes collegiality and cooperation outside the class. This spirit of cooperation furthers success since in the absence of a quota for the number of A’s and B’s given, students become motivated to work hard and to work together to improve their performance and excel.

#### 4. RESULTS OF THE EXPANDED RESEARCH

The 1998–2000 results encompass nineteen project classes taught by three instructors and nineteen control classes taught by eight instructors. There were a total of 614 students in the project classes and 648 in the control classes. Project and control classes were not flagged. Students chose their classes as appropriate to their academic or work schedule. The project and control classes were in three subjects: Elementary Algebra (Math 110), Intermediate Algebra (Math 112), and College Algebra (Math 140). All classes carried four credit hours.

Moreover, to measure the effects of the program on generalized educational skills, in particular concentration skills, we administered pre- and post-tests in College Board “Descriptive Tests in Mathematics Skills in Arithmetic Skills” and “Descriptive Tests of Language Skills in Reading Comprehension” (forms K and L) to all students.

Figure 1 presents the gains in College Board’s Arithmetic Tests for the project and the control classes. As is shown, the improvement of students in project classes was about double that of students in control classes. Since arithmetic is a prerequisite skill for all algebra classes, the improvement may be attributed, at least in part, to improved concentration skills.

Figure 2 presents the change in College Board’s Reading Comprehension Tests scores for project and the control classes. No reading or language skills were taught in the project classes. We interpret the marked

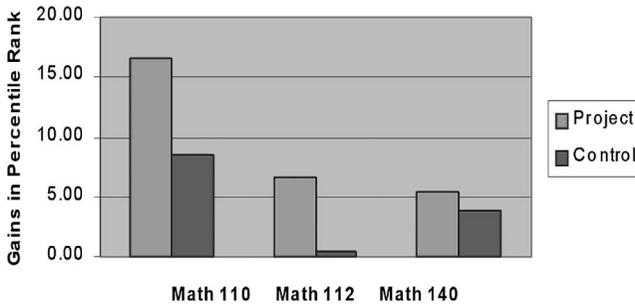
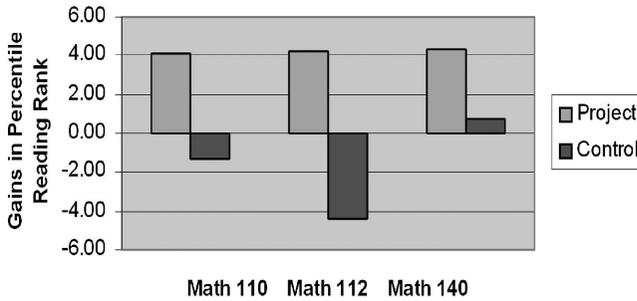


Figure 1. Gains in Arithmetic Percentile Rank 1998–2000.

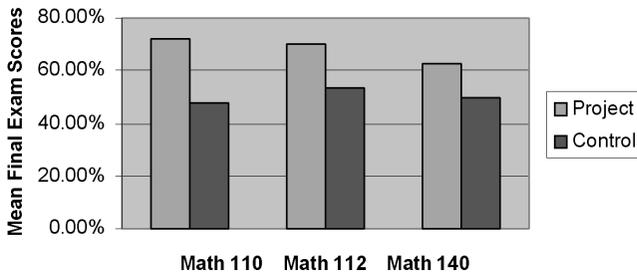


*Figure 2.* Gains in Percentile Reading Rank 1998–2000.

difference in the improvement of reading comprehension scores between students in the project classes and students in the control classes as a manifestation of improved study skills, most likely improved concentration skills. This finding is fully in line with the results reported in the previous study [19, 20]. They provide experimental verification of Plato's observation made some 2400 years ago in *The Republic*: Training in mathematics can sharpen the mind and thus produce positive results in other areas of student learning [4].

To assess the effects of the program on the learning of mathematics, we administered a departmental common final exam to project and to control classes. In Figure 3 we report the mean final exam scores for the project and control classes. Combining the results of the three courses, we see that students in the project classes scored about 18 percentage points, roughly two letter grades, higher than students in the control classes.

Another important measure of the effectiveness of an instructional program is the pass rate of students on a common instrument, such as the common final exam. In this sense, we defined the pass rate as the percentage of students who scored 70 percent or better on the common final exam, showing that they are ready to take the next mathematics class. We report the



*Figure 3.* Mean Final Exam Scores 1998–2000.

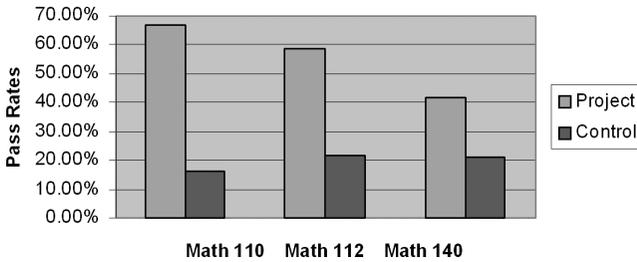


Figure 4. Pass Rates on Final Exam 1998–2000.

pass rate of students for project and control classes in Figure 4. As is seen, pass rates in the project classes are four times the control classes in Math 110, about three times the control classes in Math 112, and twice the control classes in Math 140.

We also present the section by section comparisons of the Keystone and non-Keystone classes in Math 110 and 112 for the period 2001–2003. Figure 5 presents the mean scores for the common final exam in Math 110 in fall 2001 where Keystone classes achieved the first, the second and the fifth rankings among sixteen sections.

Figure 6 presents the mean scores for the common final exam in Math 112 in fall 2001 where Keystone classes achieved the first, the second and the fourth rankings among eleven sections.

Figure 7 presents the mean scores for the common final exam in Math 110 in spring 2002 where the Keystone class achieved the highest ranking among fifteen sections.

Figure 8 presents the mean scores for the common final exam in Math 112 in spring 2002 where the Keystone class achieved the highest ranking among seven sections.

In fall 2002, there was no departmental common final exam; however, the Keystone class scored the unprecedented mean score of 82% with 22 percentile improvement in reading scores from pre-test to the post-test.

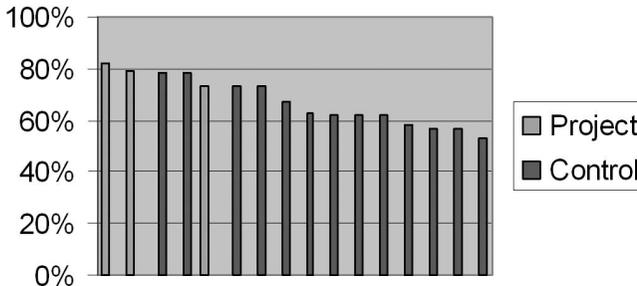


Figure 5. Math 110 Fall 2001 Mean Final Exam Scores.

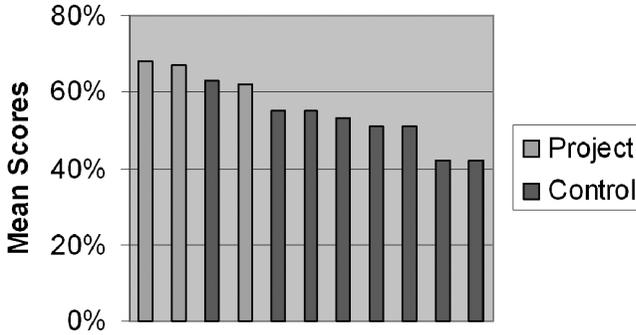


Figure 6. Math 112 Fall 2001 Mean Final Exam Scores.

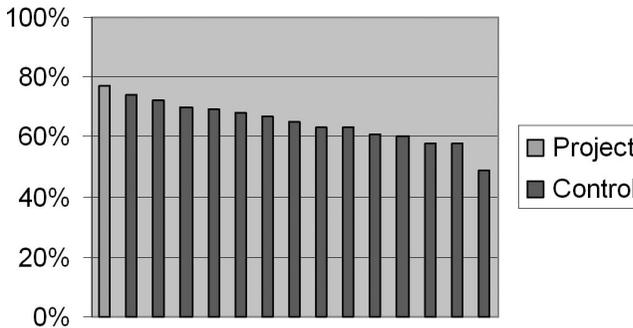


Figure 7. Math 110 Spring 2002 Mean Final Exam Scores.

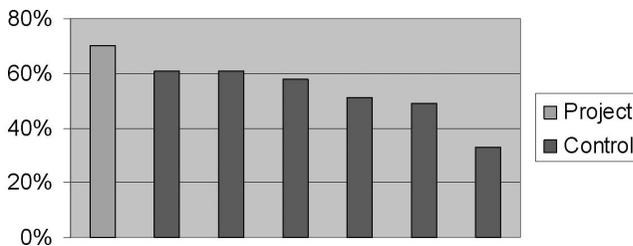


Figure 8. Math 112 Spring 2002 Mean Final Exam Scores.

In spring 2003, we once again administered a departmental final exam to all 13 Math 112 classes. As is shown in Figure 9, the Keystone class finished 12 percentage points, a full letter grade, higher than any other section.

The results reported in our study should be read in conjunction with the retention rates. It is possible to achieve high performance outcomes through the attrition of weaker students. However, our program achieved higher

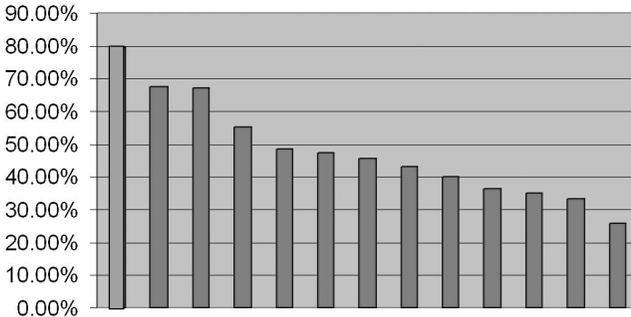


Figure 9. Math 112 Spring 2003 Mean Final Exam Scores.

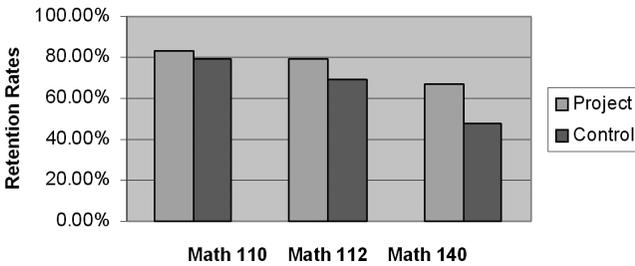


Figure 10. Retention Rates 1998–2000.

student scores at no cost to the retention. We present the retention rates for project and control classes in Figure 10. The retention figures clearly show that the higher performance was achieved with higher retention.

### 5. PERSISTENCE OF KEYSTONE STUDENTS

To study the impact of the Keystone program on students in subsequent semesters, we investigated the persistence of our Keystone and Control students in mathematics and in other disciplines at the college. Our findings show that 41% of those entering Keystone intermediate algebra classes (Math 112) in fall 2001 went on to take and pass a higher level college credit math course in the subsequent semesters. This result compares with 23% of those entering Control intermediate algebra classes who went on to take and pass a college credit math course in the future. These figures show that Keystone students' persistence in mathematics was almost twice that of the control classes.

We also studied the persistence of students in the college; that is to say the percentage of students who took at least one class in the college in the subsequent term. We found that 79% of those entering Keystone intermediate

algebra classes in fall 2001 persisted in the college. This compares with 63% of those entering intermediate algebra in the control group.

## 6. DISCUSSION

The Keystone method provides a synergistic effect in improving students' general educational skills. Improvement of such skills as concentration skills is responsible for increased outcomes in reading scores in a mathematics class. While the target population in the Keystone classes has been those in remedial mathematics classes, the benefit derived transcends the "remediation" of the subject. Acquisition of basic skills in learning contributes to the acquisition of higher order learning. This is especially true in mathematics since, due to the hierarchical nature of the subject, mastery in basic skills facilitates the attainment of higher order conceptual understanding [26]. Through teaching mathematics, the Keystone method inculcates work and study habits that could benefit students educationally, in mathematics and in other subjects. Frequent and cumulative testing, immediate feedback, requiring precision and fluency in basic skills, attainment of mastery, repeated review and practice, and adjustment of teaching according to student response, are elements of the Keystone method.

The Keystone method is a partnership between the learner and the teacher. It applies continuous monitoring of learning outcomes and adjusting of teaching practices. This constant interaction and involvement of all partners in the teaching and learning experience makes the Keystone classroom such a successful learning experience.

## 7. CONCLUSION

The Keystone Project has continued to demonstrate that mathematics can serve as a "keystone," i.e., an inter-linking block, in the education of students needing remediation. It improves the students' knowledge of the subject matter as well as improving their basic educational skills. Moreover students in classes using the Keystone method show better persistence in the discipline and at the college.

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